# Calculation of surface electric field on UHV transmission lines under lightning stroke 

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#### Abstract

Lightning is the major cause of interruptions in power transmission lines. Ultra-high voltage (UHV) AC transmission line has been put into operation in China. Due to the large tower height up to 90 m and the high operation voltage up to 1000 kV , the shielding failure probability increase obviously. Nowadays the leader progression model is an advanced method to evaluate the shielding failure probability of transmission lines. The surface electric field is the key issue for lightning failure analyzing, because the upward leader inception mainly depends on the surface electric field. In this paper, the lightning leader model is introduced. A charge simulation method based piecewise linear function is adopted to analyze the surface electric field on phase conductors and ground wires. The influence factors, such as operation voltage, lightning peak current, lightning down leader position, protection angle, on surface electric field were analyzed.


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## 1. Introduction

The shielding failure is the major cause of interruptions in power transmission systems. The rational design to improve the shielding effect of transmission line against lightning is one of the key problems of transmission line design. Now $1000-\mathrm{kV}$ ac ultra-high voltage transmission line has been put into operation in China. The transmission tower reaches about 70-90 m high, and $30-60 \mathrm{~m}$ wide, thus, the lightning attraction area of $1000-\mathrm{kV}$ transmission line is much wider. Meanwhile, with the increment of operation voltage of the long-distance transmission lines, proportion of shielding failure rises [1]. Due to the high insulation level of the $1000-\mathrm{kV}$ transmission line, the threaten of direct lightning stroke is very small, but the probability of shielding failure increases obviously due to very high towers.

According to the statistical results of power system failure classification, above $50 \%$ of power system failures were caused by lightning in Japan [1]; about 40-70\% of the total tripping numbers of transmission lines in high voltage power system were caused by lightning in China [2]. According to the operation experiences

[^0]of $1150-\mathrm{kV}$ ac transmission lines in Russia, the lightning trip-out rate was $84.4 \%$, its length is 493.2 km . The first double circuit UHV transmission line which has a length of approximate 490 km was completed in Japan in 1999, and it has being operated in 500 kV [3]. Field observation of the characteristics of direct lightning strokes to the double circuit UHV transmission line was carried out by The TEPCO during 1998-2004 [4].

The Electro-geometry Method (EGM) [5-7] treats the lightning strike process as a geometry drawing, the effect of conductor dimension cannot be considered, so for $1000-\mathrm{kV}$ transmission line with the tower height taller than 70 m , this effect would have strong influence on the upward leaders from tower, shield wires and phase conductors, certainly on the downward lightning leader, too.

With the progress of the long air gap discharge research, leader progress model (LPM) has become the new approach to analyze the shielding failure of transmission line. Many researchers had studied LPM [8-11] and proposed different models. LPM considers the variation of space electric field and its effect on the lightning developing progress during the lightning striking the conductor. It is much more approaching to the physics of lightning, is able to overcome the difficulty of EGM.

The LPM proposed a description of the whole developing process of descending leader to upward leader. An assessment model is proposed here in the conception of LPM, based on the lightning survey data and the physics of leader discharge. The LPM is an advanced


Fig. 1. Lightning leader progress model.
method for evaluating shielding failure probability of transmission lines nowadays.

It is a key issue to calculate the surface electric field on phase conductors and ground wires, because the upward leader inception mainly depends on the surface electric field. The charge simulation method has developed rapidly in the recent years and has emerged as a very efficient and accurate method for electric field calculations [12,13]. In this paper, a charge simulation method based piecewise linear function is applied. The surface electric field on different phase conductors and ground wires is analyzed. The influence of critical radius of a conductor on the surface electric field is taking into account. The influence factors, such as operation voltage, lightning peak current, lightning down leader position, protection angle, on surface electric field were analyzed.

## 2. Method for surface electric field calculation under lightning stroke

### 2.1. Method for surface electric field calculation

In the lightning LPM, the downward and upward leader, transmission line and ground are considered as charged structure. The electric fields of these objects are calculated simultaneously. When the surface electric field on an object exceeds the critical value, then an upward leader generates from this object. When the average electric field between the downward leader and upward leader tips exceeds the critical breakdown one, then the downward leader emerges with the upward leader, and lightning strikes this object [5-7]. Fig. 1 shows the lightning leader progress models.

The cloud is represented by a mono-polar surface charge model, the amount of charge is 8 C , and the diameter of the surface is 10 km [8]. For the relationship between lightning peak current $I(\mathrm{kA})$ and total charge $Q_{c}(C)$ in leader channel, different researchers gave different formulas [8,11,14,15]. Formula in [11] is significant different from others when lightning peak current is large, the others have a good agreement. In this paper, the formula in [8] is used.
$Q_{c}=76 \times 10^{-3} \cdot I^{0.68}$
Several distributions, such as uniform distribution, linear distribution, and exponential distribution are considered for total charge in lightning channel $[3,8,11]$. Observations show that the charge density in lightning channel head is quit larger than in other parts. Therefore, one reasonable model is that charge in the lightning channel is considered as uniform distribution, and concentrate charge is used to simulate the head of the lightning channel, the relationship between the concentrate charge and the charge density can be establish by Gauss' law. In this paper, the lightning


Fig. 2. Image of charge above the ground.
channel head is considered as a semi-sphere with radius $r_{s}$, and the channel is a cylinder with the same radius. Then the concentrate charge in the head and charge density in the channel can be represented as:
$q_{c}=\frac{Q_{c}}{l}$
$Q_{h}=2 \pi \varepsilon_{0} r_{s}^{2} E_{S}$
where $q_{c}$ is the charge density in the lightning channel, $C / \mathrm{m} . l$ is the lightning channel length, usually is taken as $2 \mathrm{~km} . Q_{h}$ is the concentrate charge in the head, C. $E_{S}$ is minimum field strength to keep lightning developing itself in the channel and head. For positive leader, it is $5 \mathrm{kV} / \mathrm{cm}$, for negative leader, it is $10 \mathrm{kV} / \mathrm{cm}$ [14]. The radius of the semi-sphere tip of the downward leader $r_{s}(\mathrm{~cm})$ is calculated by [5]
$r_{s}=3.0 \log (I+20)$
The electric field is treated as electrostatic field or quasielectrostatic field, when analyzing the lightning strike the objects on the ground. Lightning leader can be simulated by charge source. Therefore, it is easy to get the relationship between the lightning current and total charge in the lightning channel, and is effective on electric field calculation. In this paper, shielding failure of UHVDC is the main key topic, so charge simulation model is applied.

In the charge simulation method, the simulation charges are placed in the central axis of conductors [15,16]. In order to maintain the surface voltage potential of the conductor, charges are induced on the conductor. Every sub-conductor is represented by a linear charge. The ideal ground is represented by image charge underground. As shown in Fig. 2, the image of a point charge $-Q$ located at $(x, y, z)$ is a point charge $Q$ at $(x, y,-z)$.

For bundle conductors and fixed gap length,
$\mathbf{A} \mathbf{Q}+\mathbf{A}_{1} \mathbf{Q}_{1}=\boldsymbol{\Phi}$
where $\mathbf{Q}$ is simulation charge column matrix of sub conductors and images, $\mathbf{Q}_{1}$ is simulation charge of lightning leaders and their images. A and $\mathbf{A}_{1}$ is the corresponding voltage potential coefficient matrix, where is only determined by the positions on sub conductors, lightning leaders and their images. $\Phi$ is the voltage potential column matrix on the check points. Then, the simulation charges of the bundle conductors and their mirrors can be calculated as (6). The surface electric field also can be calculated by (7).
$\mathbf{Q}=\mathbf{A}^{-1}\left(\boldsymbol{\Phi}-\mathbf{A}_{1} \mathbf{Q}_{1}\right)$
$\mathbf{E}=\mathbf{F Q}$
where $\mathbf{F}$ is the electric field coefficient matrix which is determined by the positions of conductors and leaders. $\mathbf{E}$ is the surface electric field vector.

Considering a finite horizontal straight conductor, assume the line charge density as a piece linear function. Set the central axis


Fig. 3. Line charge density as piecewise linear function.
of the conductor as $x$ axis, and set the vertical direction of the conductor as $z$ axis. Divide a conductor into $N$ segments, source points (dots in Fig. 3.) are located at the endpoints of each segment, $q_{j}$ is the simulate charge density at source points $S_{j}$. Check point $C_{j}$ (circles in Fig. 3) are located on the surface of the conductor directly above source point. The charge density at $S, q(S)$, is calculated by the following formula:
$q(S)=q_{j} \frac{\left|\overrightarrow{S S_{j+1}}\right|}{\left|\overrightarrow{S_{j} S_{j+1}}\right|}+q_{j+1} \frac{\left|\overrightarrow{S S_{j}}\right|}{\left|\overrightarrow{S_{j+1} S_{j}}\right|}, \quad S \in \overrightarrow{S_{j} S_{j+1}}, \quad j=1,2, \ldots, N$
where $\overrightarrow{S S_{j+1}}$ represents the vector between point $S$ and $S_{j+1}$. Thus, the potential of check points, $\phi_{c_{i}}$, can be derived from the following expression:
$\phi_{C_{i}}=\sum_{j=1}^{N} \int_{S_{j}}^{S_{j+1}}\left(\frac{q(S)}{\left|\overrightarrow{S C}_{i}\right|}-\frac{\tilde{q}(S)}{\left|\vec{S}_{i}\right|}\right) d l, \quad i=1,2, \ldots, N$
where $\tilde{q}(S)$ is the image of $q(S)$, and $d l$ is the length infinitesimal element. The potential of check points is determined by the operation voltage. Solve the linear equation (9), we can get the linear charge density.

The radius of the conductor is $r_{0}$.

### 2.2. Upward leader inception criterion for bundle conductors

According to long air gap discharge theory, the electrodes’ size will influence the breakdown voltage, and the critical size exists. Under the critical size, the upward leader inception voltage is equal to the corona inception voltage [17,18]. Therefore, under the same condition, the upward leader inception electric field is equal to corona inception electric field. The conductor surface electric field criterion is that when the conductor surface electric field is larger than the corona inception electric field, the upward leader will come out. The corona inception electric field $E_{c}(\mathrm{kV} / \mathrm{m})$ can be calculated by the Peek's law [11,19]:
$E_{c}=3000 \delta m\left(1+\frac{0.03}{\sqrt{\delta r}}\right)$
where $\delta$ is relative density of air, $m$ is roughness coefficient, $r$ is the critical radius, m . Therefore, the surface electric field refers to the electric field at the critical radius of a conductor.

According to $[10,18]$, this paper calculates the critical radius of different bundles as the bundle spacing is 500 mm . For non-bundle conductor, the critical radius is 10 cm , for four-bundle conductor, the critical radius is 4 cm , which has a good agreement with the experiment result by Les Renardieres Group. The result shows that the bundle spacing has little influence on the critical radius. The critical radius is mainly determined by the bundle number (Table 1 ).

Table 1
The critical radius of different bundle conductor.

| Bundle number | 1 | 2 | 4 | 6 | 8 |
| :--- | ---: | :--- | :--- | :--- | :--- |
| Critical radius (cm) | 10 | 4 | 6 | 3 | 2 |



Fig. 4. 1000 kV AC tower (in mm).

### 2.3. Calculation parameters

The surface electric field on phase conductors and ground wires depends on a number of factors, such as the lightning peak current, the height of tower, the operation voltage of transmission lines, the protection angle and some other factors.

In the following calculation, the surface electric field of the ground wires and conductors shown in Fig. 4 under negative


Fig. 5. Surface electric field of conductor \#5 with different length of transmission lines in calculation.


Fig. 6. Surface electric field of conductor \#5 under different lightning peak currents.
lightning stroke will be calculated. The critical radius of ground wires \#1-\#2 is 0.1 m , and that of 8 -bundled conductors \#3-\#5 is 0.02 m . The length of transmission lines in calculation is 400 m . The lightning downward leader starts at the side of conductor \#5, and the lightning is set as negative according to observing results [8].

Calculation results of the surface electric field of conductor \#5 with different length of the transmission lines are shown in Fig. 5. The lightning peak current is set as 50 kA . The length of transmission lines in calculation has trivial influence on surface electric field of conductors and ground wires. The calculation results will be more accurate when the length of transmission lines is more than 400 m which is the calculation length in this paper.

## 3. Characteristics of surface electric field under lightning stroke

### 3.1. The influence of lightning peak current

Assume that the horizontal distance between downward leader and the center of lines is 50 m . Fig. 6 shows the surface electric field on conductor \#5 at different lightning peak currents. Considering the most severe condition, the voltage of conductor \#5 which is close to the cloud is at the positive peak.

Calculation results show that, when the operation voltage is considered, it is easier for upward leader to initiate when the operation voltage is considered, because more charges will be induced on conductor \#5 to maintain the peak operation voltage.


Fig. 7. Relationship between electric field and lightning peak current (neglecting the operation voltage).

As shown in Fig. 7, neglecting the operation voltage, when the heads of the downward leaders are at the same height $(H)$, the surface electric field of conductor \#5 is enhanced linearly with the lightning peak current increasing, and the upward leader can initiate earlier.


Fig. 8. Surface electric field of conductor \#5 under different horizontal distances.


Fig. 9. Surface electric field of conductor \#5 under different protection angles.

### 3.2. The horizontal distance between downward leader and the center phase

The lightning peak current is set as 50 kA . Fig. 8 shows the relationship between the surface electric field of conductor \#5 and the horizontal distance between downward leader and the center of lines. $Y$ is the horizontal distance between downward leader and the center phase. The surface electric field of conductor \#5 reduces with the increasing of the horizontal distance between downward leader and the center of lines. When the horizontal distance between downward leader and the center of lines is too large, the surface electric field on conductors will not be high enough for upward leader inception, before the downward leader strikes the ground. In other words, only within a certain distance can the shielding failure occur.

In the beginning of the leader progression process, the influence of horizontal distance between downward leader and the center of lines on surface electric field of conductor \#5 is small; however the influence becomes bigger as the leader develops. Surface electric field of the conductor is larger when operation voltage is taken into consideration.

### 3.3. Protection angle

Ground wires of transmission lines are designed to shield conductors. Comparing with conductors, ground wires are more likely to be struck by lightning.


Fig. 10. Stroke path under different protection angle.

Assume that the lightning peak current is 50 kA , and the horizontal distance between downward leader and the center of lines is 50 m . Calculation results of the surface electric field of conductor \#5 at different protection angles are shown in Fig. 9. The surface electric field of conductor \#5 increases as the protection angle increase. However, the influence of the protection angle on the surface electric field of the conductor is very small. That is because the surface electric field of the conductor is determined by the cloud charge, downward leader charge and the induced charge on the


Fig. 11. Surface electric field of conductor \#5 with different phase angle.

(a) Conductor \#5 (Neglecting operation voltage)

(b) Ground wire 2 (Neglecting operation voltage)

(c) Conductor \#5 (Considering the operation voltage)

(d) Ground wire 2 (Considering the operation voltage)

Fig. 12. Surface electric field of conductor $\# 5$ with different tower height.
conductor. Surface electric field of the conductor is larger when operation voltage is taken into consideration.

Fig. 10 shows the upward leader path under different protection angle. When the protection angle becomes smaller, the upward leader from the side conductor is more close to the head of downward leader. That is the major reason why the smaller protection angle makes a better lightning shielding performance.

### 3.4. Operation voltage of transmission lines

Assume that the lightning peak current is 50 kA , and the horizontal distance between downward leader and the center of lines is 50 m . When the head of the downward leaders is 500 m high, the surface electric field of conductor \#5 with different phase angles are shown in Fig. 11.

The phase angle of the operation voltage of the conductor has great impacts on the surface electric field of the conductor. For negative lightning, surface electric field is relatively larger when operation voltage is at the positive half circle. Due to the influence of adjacent phase, the electric field different when the instantaneous voltages are the same but phase angles are different.

More accurate shielding failure rate should be derived from the weighted average value of results under different phase angle. It is different from results neglecting operation voltage.

### 3.5. Height of the tower

Assume that the lightning peak current is 50 kA , and the horizontal distance between downward leader and the center of lines is 50 m . Calculation results of the surface electric field of conductor \#5 when the height of the tower is added by $0 \mathrm{~m}, 20 \mathrm{~m}, 50 \mathrm{~m}$ and 100 m are shown in Fig. 12.

The surface electric field of the conductor and ground wire increases as the height of the tower increases. That means the upward leader will start earlier when the tower is higher, thus the shielding effect of ground will become weaker. Therefore, shielding failure rate will be higher.

## 4. Conclusions

A charge simulation method with piecewise linear density for lightning leader progress model is presented. The phase conductors and ground wires are represented by line charges in the central axis. Cloud is represented by surface charge. Charge in the lightning channel is considered as uniform distribution, and concentrate charge is used to simulate the head of the lightning channel.

With the increase of the lightning peak current and the height of tower, the surface electric field of the conductor is enhanced. And the surface electric field of conductor increases with the decreasing of the horizontal distance between downward leader and the center of lines.

The surface electric field of the conductor under negative lightning stroke is relatively larger when operation voltage of the conductor is in the positive half cycle. Protection angle has a little influence on the surface electric field of phase conductors and ground wires.

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