Advances in Nonlinear differential equations : Analysis, Numerics and Applications

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2022. 08. 13—2022. 08. 14(China)

2022. 08. 12—2022. 08. 14(US)

Workshop Schedule

Time & Date **Beijing time**	**Saturday** **(August 13)**	**Sunday** **(August 14)**
8:00-8:50	Monica Visan	Zhenning Cai
8:50-9:40	Soonsik Kwon	Qinglin Tang
9:50-10:30	Jia Shen	Shaobo Zhang
10:40-11:30	Tran Minh Binh	
13:00-13:50		Xu Yang
20:00-20:50	Jacek Jendrej	Ruobing Bai
20:50-21:40	Gong Chen	Liang Li
21:40-22:30	Lili He	Haoyang Wu
22:30-23:20	Xiaoxu Wu	Xueyan Zhai

Zoom link：

https://zoom.us/j/9028252952?pwd=bTAvYzliS2RXQ2h3YjNCSHpSeTNxZz09

ID: 902 825 2952 ; Password: mh4x0R

Invited Talks

Determinants, commuting flows, and recent progress on completely integrable systems

Monica Visan

University of California, Los Angeles, USA

Abstract. I will survey a number of recent development in the theory of completely integrable nonlinear dispersive PDE. These include a priori bounds, the orbital stability of multisoliton solutions, and well-posedness at optimal regularity. I will describe the basic objects that tie together these disparate results, as well as the diverse ideas required for each problem.

On the blow-up dynamics of the self-dual Chern-Simons-Schrödinger equation under equivariant symmetry.

Soonsik Kwon

Korea Advanced Institute of Science and Technology, Korea

Abstract. In this talk, I will present recent joint works with Kihyun Kim and Sung-Jin Oh on the self-dual Chern-Simons-Schrödinger equation (CSS) within equivariant symmetry. CSS is a gauge-covariant 2D cubic nonlinear Schrödinger equation. It also enjoys the mass-critical scaling/pseudoconformal invariance and soliton solutions. I will discuss recent results on finite time blow-up constructions, with emphasis on an interesting rotational instability mechanism. If time allowed, I will also discuss soliton resolution for this model, which is a remarkable consequence of the self-duality and non-local nonlinearity.

Almost sure scattering for non-radial energy critical NLS

Jia Shen(申佳)

Tianjin University, China

Abstract. In this talk, I will present a recent work about the almost sure scattering for the non-radial defocusing energy critical non-linear Schrodinger equations in 3D and 4D cases. This is finished joint with Prof. Avy Soffer and Prof. Yifei Wu. Previously, in the energy critical case, there is no probabilistic global large data result for 3D NLS, and all the known probabilistic scattering results in 4D case require the initial data in H^s with some s>0. We proved the scattering for 3D and 4D defocusing energy critical NLS for almost all the non-radial data in H^s with any s in R. In particular, our result does not rely on any spherical symmetry, size or regularity restrictions.

TBA

Tran Minh Binh

Texas A&M University, USA

Abstract. TBA

Soliton resolution for the energy-critical wave maps equation in the equivariant case

Jacek Jendrej

Sorbonne Paris North University, French

Abstract. I will present a joint work with Andrew Lawrie (MIT) on the wave maps equation from the (1+2)-dimensional space to the 2-dimensional sphere, in the case of initial data having the equivariant symmetry. We prove that every solution of finite energy converges in large time to a superposition of harmonic maps (solitons) and radiation. It was proved by Côte, and Jia and Kenig, that such a decomposition is true for a sequence of times. Combining the study of the dynamics of multi-solitons by the modulation technique with the concentration-compactness method, we prove a "non-return lemma", which allows to improve the convergence for a sequence of times to convergence in continuous time.

Long-time dynamics of the sine-Gordon equation

Gong Chen (陈功)

Georgia Tech, USA

Abstract. I will discuss the soliton resolution and asymptotic stability for the sine-Gordon equation. It is known that the obstruction to the asymptotic stability for the sine-Gordon equation in the energy space is the existence of small breathers which is also closely related to the emergence of wobbling kinks. Our stability analysis gives a criterion for the weight which is sharp up to the endpoint so that the asymptotic stability holds. This is a joint work with Jiaqi Liu and Bingying Lu.

The linear stability of weakly charged and slowly rotating Kerr-Newman family of charged black holes

Lili He(何丽丽)

Johns Hopkins University, USA

Abstract. I will discuss a work in progress in which we study the linear stability of weakly charged and slowly rotating Kerr-Newman black holes under coupled gravitational and electromagnetic perturbations. We show that the solutions to the linearized Einstein-Maxwell system decay at an inverse polynomial rate to a linearized Kerr-Newman solution plus a pure gauge term. The proof uses tools from microlocal analysis and a detailed description of the resolvent of the Fourier transformed linearized gauge-fixed Einstein-Maxwell operator at low frequencies.

Large time asymptotics of Schrödinger type equations with general data

Xiaoxu Wu(吴晓旭)

Rutgers University, USA

Abstract. We consider Schrödinger equations with a general interaction term, which is linear or nonlinear, time dependent and including charge transfer potentials. Without the assumption of radial symmetry, we prove the global solutions are asymptotically given by a free wave and a weakly localized part. The proof is based on constructing in a new way the Free Channel Wave Operator, and further tools from the recent work of Baoping Liu and Avy Soffer[1,2]. Our work part of their work to arbitrary dimension, and non-radial data. Convergence rate is also provided. This new method also allows us to study Klein-Gordon equations, N-body Quantum and general 3-body systems. These are joint works with Avy Soffer.

[1]Liu, B., & Soffer, A. (2020). A General Scattering theory for Nonlinear and Non-autonomous Schrödinger Type Equations-A Brief description. arXiv preprint

[2] Liu, B., & Soffer, A. (2021) The Large Time Asymptotics of Nonlinear Multichannel Schrödinger Equations".

Submitted

High-order numerical methods for stochastic differential equations on special linear groups

Zhenning Cai (蔡振宁)

National University of Singapore, Singapore

Abstract. In the computation of path integrals in quantum chromodynamics, an important approach is the Langevin method, which requires to solve stochastic differential equations on the special unitary group or the special linear group. In such applications, the number of variables is often huge. Therefore, we are interested in numerical methods whose computational complexity depends only linearly on the number of variables. Unfortunately, for most classical numerical methods for stochastic differential equations, the computational cost is usually proportional to a certain power of the number of variables, and the power increases when the order of accuracy increases. In this talk, we consider a specific type of stochastic differential equations defined on the special linear group, and we develop a series of Runge-Kutta schemes from the half order to the second order, in which the computational cost grows only linearly with the number of stochastic functions. Our numerical tests verify both the accuracy and the efficiency of our schemes.

Numerical methods for computing the ground state and dynamics of the rotating dipolar Bose-Einstein condensate

Qinglin Tang (唐庆粦)

Sichuan University, China

Abstract. In this talk, we will present efficient numerical methods to compute the ground state and dynamics of the rotating dipolar Bose-Einstein condensate. The methods consist of three merits: (i) efficient and accurate numerical methods will be proposed to evaluate the nonlocal dipole-dipole interaction. (ii) a nonlinear conjugate gradient method, accelerated by some well-adapted preconditioners, will be developed to compute the ground states. (iii) a rotating Lagrangian coordinate transformation will be presented to eliminate the rotation term, based on which time splitting spectral methods will be presented to simulate the dynamics.

Optimal zero-padding of kernel truncation method

Shaobo Zhang (张少波)

Tianjin University, China

Abstract. The kernel truncation method (KTM) is a commonly-used algorithm to compute the convolution-type nonlocal potential $\Phi(\mathrm{x}) = (\mathrm{U} * \rho)(\mathrm{x})$, $\mathrm{x} \in \mathbb{R}^d$, where the convolution kernel $\mathrm{U}(\mathrm{x})$ might be singular at the origin and/or far-field and the density $\rho(\mathrm{x})$ is smooth and fast-decaying. In order to capture the oscillations of the Fourier integral, one needs to carry out a zero-padding of the density. In this talk, we present the optimal zero-padding factor, $\sqrt{d}+1$, for the first time together with a rigorous proof, which helps make the computational time and memory cost greatly reduced. Then, we present the error estimates of the potential in d dimension and re-investigate the optimal zero-padding factor for the anisotropic density. Extensive numerical results are provided to confirm the accuracy, efficiency, optimal zero-padding factor for the anisotropic density, together with some applications to different types of nonlocal potential.

Machine Learning and Seismic Tomography

Xu Yang (杨旭)

University of California，Santa Barbara, USA

Abstract. The stochastic gradient descent (SGD) method and deep neural networks (DNN) are two main workhorses in machine learning. In this talk, we present some preliminary results on connecting SGD and DNN to the applications in seismic tomography. On the one hand, motivated by SGD, we propose to use random batch methods to construct the gradient for iterations in seismic tomography. On the other hand, we use deep neural networks to create a reliable PmP database from massive seismic data and study the case in Southern California. The major difficulty lies in that the identifiable PmP waves are rare, making the problem of identifying the PmP waves from a massive seismic database inherently unbalanced.

Large global solutions for energy-critical nonlinear Schrödinger equation

Ruobing Bai(白若冰)

Tianjin University, China

Abstract. In this talk, we consider the global well-posedness and scattering for the 3D defocusing energy-critical nonlinear Schrödinger equation in $H^s, s < 1$. Applying the outgoing and incoming decomposition presented in the previous Beceanu-Deng-Soffer-Wu's work, we show that any radial function f which is in $H^s, \frac{5}{6} < s < 1$ outside the unit ball, there exists an outgoing component f^+ (or incoming component f^-) of f, such that when the initial data is f^+, then the corresponding solution is globally well-posed and scatters forward in time; when the initial data is f^-, then the corresponding solution is globally well-posed and scatters backward in time.

Energy distribution of solutions to defocusing semi-linear wave equation in sub-conformal case

Liang Li(李亮)

Tianjin University, China

Abstract. We consider a semi-linear, defocusing wave equation in sub-conformal case in the higher dimensional space whose initial data are radical and come with a finite energy. We prove some decay estimates of the the solutions if initial data decay at a certain rate as the spatial variable tends to infinity. A combination of this property with a method of characteristic lines give a scattering result if the initial data satisfy a decay condition.

Prediction of the lifetime of localized turbulence in channel flow using revolutional neural networks

Haoyang Wu(伍昊洋), Baofang Song(宋保方)

Tianjin University, China

Abstract. At transitional Reynolds numbers, localized turbulence in channel flow forms large-scale banded structure, the so called turbulent band. Recent studies reported that a single turbulent band is not sustained but has a transient nature, i.e. a finite lifetime. Aside from increasing with the Reynolds number, the lifetime also increases with the band length. It is important to study this size-dependent lifetime for understanding the transition mechanism of channel flow. However, the dyanmics of a turbulent band occurs on very large time and length scales, i.e. the length of a band and the lifetime both can be very large, posing difficulties on statistical studies of the stochastic lifetime in both experiments and direct numerical simulations. In order to reduce the cost, other more efficient approaches are desired. In light of that the power of deep learning in various nonlinear problems, we try to predict the statistics of the lifetime by utilizing revolutional neural networks (RNN). Here we will show some preliminary results on reproducing the transient dynamics of turbulent bands using RNN.

The linear instability of channel flow with microgroove-type superhydrophobic surfaces

Xueyan Zhai(翟雪艳), Baofang Song(宋保方)

Tianjin University, China

Abstract. Inducing flow instability is one of the approahces to increase momentum and energy transport as well as mixing efficiency in low Reynolds number flows. It has been reported in channel flow that the anisotropic velocity slip at the channel walls caused by mocrogroove-type superhydrophobic surfaces can reduce the critical Reynolds number for the onset of flow instability, and the flow is much more unstable when only one channel wall is slippery compared to the two slippery wall setting. Here, by eigenvalue calculation and time-stepping the Navier-Stokes equations, we revisited this problem, where the Navier-Stokes equations are solved using Chebyshev collocation and Fourier spectral methods with an influence matrix techniuqe to deal with the incompressibility condition and coupled velocities through the boundary condition. Our results show the opposite, i.e. the flow with two slippery channel walls is nearly always more unstable than the one slippery wall setting. Besides, we show that further lower critical Reynolds numbers can be achieved by enhancing the anisotropy, and the microgrooves on the two channel walls have to be parallel to each other for obtaining the lowest critical Reynolds number. The results are informative for desinging wall textures to enhance transport and mixing efficienty in low Reynolds number flows.